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IDENTIFICATION AND ESTIMATION OF A CLASS OF HOUSEHOLD PRODUCTION MODELS

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SUMMARY

We consider a class of household production models characterized by a dichotomy property. In these models the amount of time spent on household production does not depend on the household utility function, conditional on household members having a paid job. We analyse the (non-parametric) identifiability of the production function and the so-called jointness function (a function describing which part of household production time is counted as pure leisure). It is shown that the models are identified in the two-adult case, but not in the single-adult case. We present an empirical application to Swedish time-allocation data. The estimates satisfy regularity conditions that were violated in previous studies and pass various specification tests. For this data set we find that male and female home production time are q-substitutes. Copyright © 2003 John Wiley & Sons, Ltd.

1. INTRODUCTION

For several of the issues that motivate the analysis of the allocation of time within households, a purely descriptive, reduced-form analysis of time-allocation data does not suffice. Examples are the evaluation of the output of home production in an economy and the computation of the value of time for individuals who do not have a paid job.[1] A microeconomic framework within which this kind of decisions can be formalized—the household production approach—has been available for a long time (Becker, 1965; Lancaster, 1966). In these models the preferences of a household are not defined in terms of quantities of goods and non-labour time, but rather in terms of activities or household products that are produced with the aid of these goods and time endowments. As a flexible model for the micro-economic decision process this approach has proved to be of great value, in particular in theoretical analyses.[2]

The implicit need for a detailed description of activities and the introduction of production functions, about the outputs of which typically no information is available, have hampered empirical applications of this framework. With only slightly more detailed information about time uses, the researcher now has to identify a two-layered nested utility function. More specifically, we

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[1] The value of the output of home production can be estimated by the total value of the production factors used (*input approach*). Normally, however, we do not have information about all of these inputs (e.g. auxiliary goods). But even if we could measure all the input quantities, this approach would systematically ignore any returns to scale and therefore the value added in home production. Fitzgerald *et al.* (1996) evaluated the value of home production by measuring output in physical units and attributing market prices to them (*output approach*.) Since households choose not to purchase these market alternatives, this approach only provides an upper bound to the value of home production.

[2] In a recent theoretical analysis Apps and Rees (1997) and Chiappori (1997) stress the importance of household production in analysing intrahousehold behaviour.

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only observe some combination of inputs (time and goods) and know that it was used to produce a set of (latent) activities that was preferred over all other sets of activities that could have been produced by feasible combinations of inputs. For an empirical analysis one is forced to limit the level of detail in which the activities are defined and one is bound to impose further restrictions in order to be able to distinguish the role of the production structure from that of the preferences.

One way of doing this was proposed by Gronau (1977, 1980). The central assumption in his articles is that home-produced commodities and market goods are perfect substitutes. This assumption and the high degree of aggregation of activities considerably simplify an empirical analysis and lead to explicit testable predictions. Pollak and Wachter (1975) argued that people don't always spend their time exclusively on one activity at a time. Even in the highly aggregated Gronau model it is likely to be the case that the time people spend on home production activities is also partially considered as leisure. Graham and Green (1984) estimated a version of the Gronau model in which they allowed for this type of *joint production*.

The assumption of perfect substitutability between market goods and home products is obviously a strong one. But with the extension of joint production the model comes much closer to a realistic description of home production decisions. As an example, consider a parent whose hourly wage exceeds the market price of one hour of child care. In the Gronau model the parent has two options to provide for child care. The first option is to take care of the child herself at home. The second option is to earn money in her job and purchase child care on the market. In the model without jointness, the parent chooses the latter option. In the model with jointness, however, she will choose to take care of the child herself if a sufficiently large share of the time spent on this activity is valued as leisure. While the jointness extension is an attractive one, it also raises the question of identifiability once again. Moreover, earlier estimates of this type of models were not very promising.[3]

In this paper we reconsider the dichotomous household production model with joint production. In Section 2 we analyse its (non-parametric) identifiability. It is shown that the models are identified in the two-adult case, but not in the single-adult case. In Section 3 we consider a specific parametric form for the production technology and joint production. Section 4 discusses an empirical application to Swedish time-allocation data. The estimates satisfy the regularity conditions of utility maximization, that were violated in previous studies. Simulation results and specification tests are presented, as well as estimates for single adult households. Section 5 concludes.

2. IDENTIFICATION

2.1. The Gronau Model

In the Gronau model the general framework of Becker (1965) is simplified as follows. In the utility function three types of arguments are distinguished (cf. the activities in Becker's model):

[3] Graham and Green (1984) used a Cobb–Douglas specification of the household production function for two-adult households, along with a specification of the jointness functions that allows the household's optimization problem to be solved analytically. None of the alternative sets of estimates Graham and Green computed for this specification made sense: either the production function itself or the jointness functions were *decreasing* functions of home production time. In Kerkhofs (1999) a slight modification of the Graham and Green specification was proposed. Application of that model to Swedish time-allocation data resulted in estimates that have the right signs, but still fail to satisfy the second order conditions for utility maximization. Newman and Gertler (1994) use Gronau's specification for households with arbitrary numbers of members. The resulting specification is incoherent in the sense that there is no production function from which these marginal production functions can be derived.

consumption goods, commodities produced at home and leisure of each of the household members.[4] The consumption goods enter the utility function as a Hicksian composite good. This commodity can be considered as *total consumption expenditures* and will be denoted by X_M. The output of home production will also be modelled as a composite good. This commodity, denoted by Z, corresponds closest to Becker's concept of an activity: it is produced by combining an amount of auxiliary goods (X_Z) with home production time (H_m and H_f, for the male and the female partner respectively). The time people do not spend on work, be it at home or in a paid job, is considered to be (pure) leisure. The key assumption in this model is that consumption goods X_M and the output of household production Z are perfect substitutes. We can therefore write the utility function as follows

$$U(X_M + Z, L_m, L_f), \tag{1}$$

where L_m and L_f are leisure enjoyed by the male and female partner respectively. U is assumed to be strictly increasing, differentiable and quasi-concave. Efficient production is characterized by the household production function

$$Z = Z(H_m, H_f, X_Z) \tag{2}$$

Z is assumed to be a monotonically increasing and twice differentiable function of the production factors. The optimal choice of purchases and time uses will be subject to the household budget constraint:

$$X_M + X_Z = V + W_m N_m + W_f N_f \tag{3}$$

Here W_i is the net wage rate of partner i and N_i is the number of hours he or she works in a paid job; V is non-labour income. The price level of the composite good is normalized at one. Finally, a feasible allocation must satisfy the time constraints

$$H_i + L_i + N_i = T, \quad i = m, f \tag{4}$$

T being the (daily) time endowment.

The household's decision problem can now be formalized as follows: the household members maximize their joint utility as defined in (1), subject to (2), (3), (4) and non-negativity constraints on X_M, X_Z and L_i, N_i, and $H_i (i = m, f)$. We will assume that by the choice of the utility function and the production function none of the inequalities will be binding in an optimal allocation, except those of N_m and N_f. This assumption is made for every combination of wage rates, non-labour income and other exogenous variables that we consider of interest. Moreover, it will be assumed that for each of these combinations the optimum is unique. The solution to the household's decision problem then has to satisfy the following Kuhn–Tucker conditions (ξ_m and ξ_f are the shadow prices of the inequality constraints on labour time; restrictions (2), (3) and (4) have been substituted into the utility function):

$$\frac{\partial Z}{\partial X_Z} = 1 \tag{5}$$

[4] We will model only the behaviour of the head of the household and—if present—his or her partner. The presence of children or any other persons in the household will be treated as exogenous in the model. For this analysis we will take the two-adult family as the standard case.

$$\frac{\partial U}{\partial Z}\frac{\partial Z}{\partial H_m} = \frac{\partial U}{\partial L_m} = \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial X_Z}W_m + \xi_m \tag{6}$$

$$\frac{\partial U}{\partial Z}\frac{\partial Z}{\partial H_f} = \frac{\partial U}{\partial L_f} = \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial X_Z}W_f + \xi_f \tag{7}$$

$$\xi_m N_m = \xi_f N_f = 0 \tag{8}$$

$$\xi_m \geq 0, \xi_f \geq 0, N_m \geq 0, N_f \geq 0$$

If in the optimum both partners participate in the labour force ($N_m > 0, N_f > 0$ and $\xi_m = \xi_f = 0$) equations (5) to (8) imply:

$$\frac{\partial Z}{\partial H_i}(H_m, H_f, X_Z) = W_i, \quad i = m, f \tag{9}$$

Equations (9) and (5) mirror the fundamental dichotomy in the Gronau model. Note that the assumption that the net marginal wage rate is (perceived to be) constant is vital for the dichotomy property to hold. Otherwise, N_m or N_f will enter the right-hand sides of (9).[5] For two-earner households the model implies a two-stage decision structure, in which the production decisions constitute the first stage and the remaining decisions—the allocation of non-production time and the purchase of consumption goods—are made in the second stage. In deriving a sub-model that describes the household production decision we therefore only need to specify the production function—the utility function only figures in the second stage. It must be noted, however, that the labour force participation decisions do also involve the utility function. This is important for econometric applications of the first-stage sub-model ((5) and (9)). This model only applies to the non-random sub-sample of two-earner families. For a structural model that would incorporate this endogenous sample stratification, the utility function has to be specified.

2.2. Joint Production

In this relatively aggregated setting possibilities for joint production between activities are mostly contained in the production function. One potentially important type of joint production is still excluded in this model: household production time may generate psychic income (satisfaction) over and above that of paid work (which is already accounted for in the relative value of leisure as modelled in the utility function). An extension that fits into the Gronau framework was suggested by Graham and Green (1984). Their specification can be interpreted as follows: if a person spends H_i hours on home production, he or she values $g_i(H_i)$ as a perfect substitute of leisure. Following

[5] For someone with a paid job, the existence of a unique interior optimum implies that the production function has to be strictly concave in terms of his or her time input and in X_Z, at the optimal allocation. As a result, an increase in one's wage rate will lead to a reduction of the number of hours spent on home production activities. If the time inputs are complements in home production, the partner's home production time will also be reduced. The reverse holds if the time inputs are substitutes. With regard to changes in non-labour income, this model predicts the absence of an income effect in the home production decisions of working couples. An increase of the non-labour income will—if leisure is a normal good—reduce both partners' number of hours worked in a job. But, provided both stay in the labour force, H_m, H_f and X_Z will remain unchanged. An overall increase in household productivity leads to an incipient increase in the value of time. For individuals with a job, this will lead to a reduction in labour time and an increase in both leisure and home production time. When it is no longer optimal to participate in the labour force, a further rise in productivity might even lead to a decrease in household production time, provided the income effect is sufficiently strong. Productivity rises will therefore only increase the marginal value of time for individuals that do not participate in the labour force.

Graham and Green we assume that the *jointness function* $g_i(.)$ is increasing, twice differentiable and concave in H_i and furthermore that $g_i'(.) \leq 1$ and $\lim_{H\uparrow T} g_i'(H) = 0$.[6] The direct utility function (1) is thus replaced by:

$$U(X_M + Z, L_m + g_m(H_m), L_f + g_f(H_f)) \tag{10}$$

It is easy to see that the dichotomy property survives. The partial optimization problem for the household production decisions of working couples now reads:

$$\max_{0\leq H_m, H_f \leq T; X_Z \geq 0} Z(H_m, H_f, X_Z) + W_m g_m(H_m) + W_f g_f(H_f) - W_m H_m - W_f H_f - X_Z \tag{11}$$

The first-order conditions are:

$$\frac{\partial Z}{\partial X_Z}(H_m, H_f, X_Z) = 1 \tag{12}$$

$$\frac{\partial Z}{\partial H_m}(H_m, H_f, X_Z) = W_m(1 - g_m'(H_m)) \tag{13}$$

$$\frac{\partial Z}{\partial H_f}(H_m, H_f, X_Z) = W_f(1 - g_f'(H_f)) \tag{14}$$

For a solution of these equations to be a local maximum, strict concavity of Z is still sufficient, but it is no longer necessary. In fact, some increasing returns to scale of household production time is allowed, provided that the extent to which this time is perceived as leisure falls more. In line with the assumptions we made for the model without joint production, we assume Z, g_m and g_f are such that for each pair of wage rates that we consider, a unique solution to maximization problem (11) exists. This solution is assumed to be an interior point of $[0, T] \times [0, T] \times \mathbb{R}_+$. Clearly, therefore, the objective function in (11) has to be (locally) strictly concave in the optimal allocation.

2.3. Identification in the Two-adult Case

Given that we usually do not observe the output of home production, the question arises whether the model can be identified on the basis of time-allocation data only. More specifically, the question is whether or not the model contains sub-classes of models that are observationally equivalent if estimation is based on (12), (13) and (14) or on reduced-form equations derived from these. By introducing jointness functions into the model, we reintroduce elements of the utility function and with it, potential identification problems. In order to know what significance can be attributed to estimated household production levels and whether a methodology based on choosing more flexible functional forms makes sense, we have to investigate what parts of this model can be identified non-parametrically on the basis of equations (12), (13) and (14), using data on H_m, H_f and—possibly—X_Z.

The first thing to notice is that we only observe allocations, that are optimal for some input vector (W_m, W_f, V). The relationships we want to estimate may thus be unobservable on parts of the choice set. To make this point more precise, we have to start by specifying what (W_m, W_f, V) can

[6] Graham and Green also assume $\lim_{H\downarrow 0} g_i'(H) = 1$. Moreover, they require g_i to be strictly concave.

be observed. Input vectors may be unobservable for several reasons. There might be institutional restrictions, like minimum wage legislation. Wage rates are likely to be bounded above by a person's productivity. Furthermore, as we restrict our attention to households in which both partners have a paid job, extremely low values of W_m or W_f and high values of V will not be present in the sample. Define the set of input vectors (W_m, W_f, V) that are observable to be Ω.[7] Non-labour income influences the labour force participation decision, but given that both partners are employed, V has no further effect on the household production decisions. We will therefore usually refer to pairs of wage rates rather than to elements of Ω. For that reason we define the set of observable wage rates as $\mathcal{W} = \{(W_m, W_f) \in \mathbb{R}^2_+ | (W_m, W_f, V) \in \Omega$, for some $V \in \mathbb{R}\}$. By the assumptions made before, we can attribute a unique optimal allocation (H^*_m, H^*_f, X^*_Z) to each input vector in $\mathcal{W}$. Now $\mathcal{H}$ is defined as the set of pairs (H_m, H_f) that are optimal for some input vector from Ω. This step may involve a further reduction in what we are able to observe, among other things due to non-concavities in Z. We will assume that $\mathcal{W}$ is a connected subset of $\mathbb{R}^2_+$. Given the assumptions about Z, g_m and g_f, made before, $\mathcal{H}$ is a connected subset of $[0, T] \times [0, T]$.

If we could observe the output of home production, the identification problem would be that observations of the production function are limited to $\mathcal{H}$. This type of identification problem is not specific for the household production model, but is in fact common to almost every structural micro-econometric model. In practice, however, we do not have information about the output of home production, so that we can only 'observe' the first-order conditions. Even in the absence of joint production we cannot observe the relationship between production factors and output directly, but we can only scan the curvature of the production function by varying the wage rates over $\mathcal{W}$. Moreover, as the price of auxiliary goods is normalized at one, the information we have, can at most describe this curvature along a two-dimensional non-linear variety in (H_m, H_f, X_Z)-space.

For each (H_m, H_f), the values for X_Z that could make this an optimal choice at some $W \in \mathcal{W}$, have to satisfy (12). This means that X_Z will be chosen such as to maximize the net production given H_m and H_f:

$$\tilde{Z}(H_m, H_f) \stackrel{\text{def}}{=} \max_{X_Z \geq 0} Z(H_m, H_f, X_Z) - X_Z \tag{15}$$

It will be assumed that for all elements of the closure of $\mathcal{H}$ this maximization problem has a unique finite, non-zero solution.[8] We will refer to $\tilde{Z}$ as the *net product value function*. The choice of X_Z will be denoted by $\psi_Z(H_m, H_f)$. In order to ensure that ψ_Z is differentiable on the closure of $\mathcal{H}$, we will also assume that $\partial Z / \partial X_Z$ is continuously differentiable.[9]

Time-allocation surveys usually contain information on H_m and H_f, but hardly ever on the amount of auxiliary goods that is used in home production activities. If no information about X_Z is available, it is clear that we can, at most, identify the net product value function $\tilde{Z}$ on $\mathcal{H}$. Equations (13) and (14) can then be replaced by

$$\frac{\partial \tilde{Z}}{\partial H_s}(H_m, H_f) = W_s(1 - g'_s(H_s)), \quad s = m, f \tag{16}$$

[7] Starting from a sampling distribution for (W_m, W_f, V) on $\mathbb{R}^3_+$, Ω can be defined as the support of this distribution.

[8] A sufficient condition is that the marginal product of auxiliary goods is a strictly decreasing function of X_Z is greater than one for $X_Z = 0$ and eventually falls below one when X_Z is increased (for every $(H_m, H_f) \in \mathcal{H}$).

[9] Given the assumptions we made before, differentiability of ψ_Z follows from the Implicit Function Theorem.

In our discussion of identifiability we ignore sampling uncertainty and treat the relationships about which we have observations as perfectly known on the set of observable combinations of W_m, W_f, V, H_m, H_f and X_Z. For any given household production model (Z, g_m, g_f) observational equivalence with another specification (F, k_m, k_f) requires the first-order conditions to be identical, i.e.[10]

$$\frac{\partial Z}{\partial H_m}(H_m, H_f, \psi_Z(H_m, H_f)) = \frac{1 - g_m'(H_m)}{1 - k_m'(H_m)} \frac{\partial F}{\partial H_m}(H_m, H_f, \psi_F(H_m, H_f)) \tag{17}$$

$$\frac{\partial Z}{\partial H_f}(H_m, H_f, \psi_Z(H_m, H_f)) = \frac{1 - g_f'(H_f)}{1 - k_f'(H_f)} \frac{\partial F}{\partial H_f}(H_m, H_f, \psi_F(H_m, H_f)) \tag{18}$$

for all $(H_m, H_f) \in \mathcal{H}$. If the values of X_Z are also observed we must also have:

$$\psi_Z(H_m, H_f) = \psi_F(H_m, H_f) \tag{19}$$

for all $(H_m, H_f) \in \mathcal{H}$. These equations guarantee that for each observable pair of wage rates, the same values for H_m, H_f and—if observed—X_Z satisfy the respective first-order conditions. From (17) and (18) it can be seen that the ability to separate the roles of the jointness functions and the production function in the observed relationships draws upon the fact that the jointness functions are individual-specific, while the production function also contains cross-effects. Accordingly, it is important that one partner's optimal choice of H_i $(i = m, f)$ is influenced by a change in the wage rate of the other partner, except possibly on a subset of $\mathcal{W}$ of Lebesgue measure zero. We will therefore assume that:

$$\begin{aligned}&\frac{\partial^2 Z}{\partial H_m \partial X_Z}(H_m, H_f, \psi_Z(H_m, H_f)) \frac{\partial \psi_Z}{\partial H_f}(H_m, H_f) \\ &\quad + \frac{\partial^2 Z}{\partial H_m \partial H_f}(H_m, H_f, \psi_Z(H_m, H_f)) \neq 0 \text{ a.e. on } \mathcal{H}\end{aligned} \tag{20}$$

This assumption establishes that the subset of $\mathcal{H}$ on which the derivative of the left-hand side of (17) with respect to H_f vanishes has an empty interior.

The next proposition states that it is in general possible to identify the presence of joint production in a model for two adult households. Only if the jointness functions for both partners are identical and linear can an observationally equivalent model without joint production exist. On the other hand, the proposition implies that the specific functional form of the joint production is not completely identified non-parametrically; even if X_Z is observed. In general there exist one dimensional equivalence classes of observationally equivalent specifications. These implications will be elaborated in the four points following the proposition. It stands to reason that we restrict our attention to models for household production in which the production function and jointness functions satisfy the regularity conditions (monotonicity, differentiability etc.) we imposed above. We will refer to these models as being *admissible*.

[10] By the assumption that for every $W \in \mathcal{W}$ the household's utility maximization problem has a unique interior optimum, the slopes of the jointness functions are less than one in optimal allocations.

Proposition 1 Given an admissible household production model characterized by a production function Z and jointness functions g_m and g_f, there exists an admissible model characterized by F, k_m and k_f that satisfies (17), (18) and (19) on $\mathcal{H}$, only if for some non-negative constant ϑ

$$\frac{1 - g'_m(H_m)}{1 - k'_m(H_m)} = \frac{1 - g'_f(H_f)}{1 - k'_f(H_f)} = \vartheta \tag{21}$$

for almost every $(H_m, H_f) \in \mathcal{H}$.[11]

Proof: In the Appendix.

The implications of this proposition for identification in the extended Gronau model will be discussed in the following four points.

(1) What does this proposition tell us about the ability to determine whether joint production is present? Suppose we cannot distinguish a given model from some alternative specification without joint production. In that case (21) becomes: $1 - g'_m(H_m) = 1 - g'_f(H_f) = \vartheta$ on $\breve{\mathcal{H}}$. This implies that the only case in which we cannot conclude whether or not there is joint production, is when the jointness functions of both partners are linear and have identical slopes on $\breve{\mathcal{H}}$. Generically, therefore, the *presence* of joint production is identified.

(2) Although identification of the presence of joint production is generically possible, the specific functional forms are in general not completely identified. For a given household production model (Z, g_m, g_f), Proposition 1 can be used to construct a range of observationally equivalent models, indexed by ϑ. From (21) we can derive

$$k_i(H_i) = \frac{\vartheta - 1}{\vartheta} H_i + \frac{g_i(H_i)}{\vartheta} \quad i = m, f \tag{22}$$

Provided $\vartheta \geq 1 - \min\{\inf_{\breve{\mathcal{H}}} g'_m(H_m), \inf_{\breve{\mathcal{H}}} g'_f(H_f)\}$, these functions satisfy all the conditions we imposed on jointness functions, except for the boundary condition $\lim_{H \uparrow T} k'(H) = 0$. As long as $\breve{\mathcal{H}}$ does not contain combinations (H_m, H_f), for which either of the two variables takes on values arbitrarily close to T, the jointness functions are not identified in the neighbourhood of T, and the definitions above can be adapted arbitrarily so as to satisfy the boundary condition. A specification of F that would then satisfy the equivalence relationships (17), (18) and (19) is:

$$F(H_m, H_f, X_Z) \stackrel{\text{def}}{=} Z(H_m, H_f, X_Z) + \frac{1 - \vartheta}{\vartheta}\{Z(H_m, H_f, \psi_Z(H_m, H_f)) - \psi_Z(H_m, H_f)\} \tag{23}$$

This specification is well-defined and admissible on $\mathcal{H} \times \mathbb{R}_+$ and can in general be extended to an admissible specification on $[0, T] \times [0, T] \times \mathbb{R}_+$.[12] If X_Z is not observed, the choice of F is much easier. Take, for example:

$$F(H_m, H_f, X_Z) \stackrel{\text{def}}{=} \frac{1}{\vartheta} Z(H_m, H_f, \vartheta X_Z) \tag{23'}$$

[11] More precisely: For every $(H_m, H_f) \in \breve{\mathcal{H}}$ where $\breve{\mathcal{H}} \stackrel{\text{def}}{=} \{h \in \mathcal{H} |\ \text{span}\ (\cap_{n=1}^{\infty} \overline{\text{cone}}\{x \in \mathbb{R}^2 | h + x \in \mathcal{H} \text{ and } \|x\| < n^{-1}\}) = \mathbb{R}^2\}$, i.e. $\breve{\mathcal{H}}$ is the set of elements of $\mathcal{H}$ from which we can move along at least two paths in $\mathcal{H}$ with linearly independent directional derivatives in the starting point. This local property is necessary to be able to differentiate equivalences on $\mathcal{H}$.
[12] If we would restrict our attention to globally strictly concave production functions, it is easy to show that whenever Z is strictly concave, so is F (on $\mathcal{H} \times \mathbb{R}_+$).

The existence of observationally equivalent specifications may seem to be slightly disappointing, but in fact the identification is far stronger than might have been expected. Having a sufficiently large amount of data at our disposal, we will be able to identify (g_m, g_f) non-parametrically within the class of admissible pairs of jointness functions on $\mathcal{H}$, up to one degree of freedom. An instance in which we may even get rid of this last degree of freedom is discussed in the next point.

(3) If the closure of $\breve{\mathcal{H}}$ contains any points for which $H_m = T$ or $H_f = T$, i.e. if we can observe values for H_m or H_f that are arbitrarily close to T (non-parametric), identification on $\breve{\mathcal{H}}$ is established, because

$$\vartheta = \lim_{H \uparrow T} \frac{1 - g_i'(H)}{1 - k_i'(H)} = 1 \quad \text{for} \quad i = m \text{ or } i = f \tag{24}$$

implying $g_m'(H_m) = k_m'(H_m)$ and $g_f'(H_f) = k_f'(H_f)$, for $(H_m, H_f) \in \breve{\mathcal{H}}$.

(4) If we restrict the jointness functions to be elements of some parametric class of functions, they are identified if no other functions (k_m, k_f) in that class satisfy (21) for some $\vartheta \neq 1$. As an application of this result it is easy to verify that the jointness functions that are used in the next section are identified within that parametric class of specifications.

2.4. Identification in the Single-adult Case

Identification of joint production in models for one adult households is much weaker. In the following proposition it will be shown that the presence of joint production is not (non-parametrically) identified, even if observations on X_Z are available. Define $\mathcal{W}$ and $\mathcal{H}$ by analogy to the two-adult case. Necessary conditions for a model with home production function Z and jointness function g to be equivalent with a model with production function F, but *no* joint production, is that the following conditions are satisfied on $\mathcal{H}$:

$$\frac{\partial Z}{\partial H}(H, \psi_Z(H)) = (1 - g'(H)) \frac{\partial F}{\partial H}(H, \psi_F(H)) \tag{25}$$

$$\psi_Z(H) = \psi_F(H) \tag{26}$$

$\mathcal{H}$ now is an interval. Define $H_0 = \inf \mathcal{H}$ and $H^* = \sup \mathcal{H}$. We can then derive the following result.

Proposition 2 Let an admissible one adult household production model be characterized by a production function Z and a jointness function g. If the set of observable wage rates $\mathcal{W}$ is bounded above, the production function F defined by

$$\begin{aligned} F(H, X) = Z(H, X) &+ \frac{g'(H_0)}{1 - g'(H_0)} \frac{\partial Z}{\partial H}(H_0, \psi_Z(H_0)) \min(H_0, H) \\ &+ \int_{\min(H_0, H)}^{\min(H, H^*)} \frac{g'(\tilde{H})}{1 - g'(\tilde{H})} \frac{\partial Z}{\partial H}(\tilde{H}, \psi_Z(\tilde{H})) d\tilde{H} \\ &+ \frac{g'(H^*)}{1 - g'(H^*)} \frac{\partial Z}{\partial H}(H^*, \psi_Z(H^*)) \max(0, H - H^*) \end{aligned} \tag{27}$$

satisfies (25) and (26) and is admissible. If Z is strictly concave on $[0, T] \times [0, T] \times \mathbb{R}_+$, so is F.

Proof: In the Appendix.

As it is this simple to find an observationally equivalent model without joint production, it will be even easier to find arbitrary equivalent models with joint production. In fact, given a model (Z, g), for every jointness function k with $k' < g'$ on [0, T], an admissible production function that makes (F, k) observationally equivalent to (Z, g), is given by (27) with $g'/(1-g')$ replaced by $(g'-k')/(1-g')$. It may be noted that for some specifications—among which the one discussed in the next section—the integral in (27) still exists when the interval of integration is replaced by [0, T]. In such cases $\mathcal{W}$ does not have to be bounded and H_0 and H^* can be set equal to zero and T, respectively. If we cannot observe X_Z, we get even more freedom in choosing observationally equivalent specifications. We must therefore conclude that this class of models has limited power for the analysis of time allocation data of one adult households in the presence of joint production. Estimation results from these models have to be considered with care.

3. SPECIFICATION

As we have no information about the auxiliary goods X_Z we will start from a specification of the net product value function $\tilde{Z}$ instead of the production function itself:[13]

$$\tilde{Z} = b_m H_m + b_f H_f + 1/2\ c_{mm} H_m^2 + 1/2\ c_{ff} H_f^2 + c_{mf} H_m H_f \tag{28}$$

The quadratic specification has the advantage that time inputs may be complements as well as substitutes in home production. If $c_{mf} < 0 H_m$ and H_f are q-substitutes, otherwise they are q-complements. Define:

$$b = \begin{pmatrix} b_m \\ b_f \end{pmatrix} \text{ and } C = \begin{pmatrix} c_{mm} & c_{mf} \\ c_{mf} & c_{ff} \end{pmatrix}$$

b is assumed to be strictly positive and C is a negative definite matrix. Heterogeneity is introduced by letting b_m and b_f depend on household specific and individual specific characteristics:

$$\log(b_s) = x_s'\beta_s + u_s \quad s = m, f \tag{29}$$

where

$$\begin{pmatrix} u_m \\ u_f \end{pmatrix} | x_m, x_f \sim \mathcal{N}(0, \Sigma_{uu})$$

These random errors account for unobserved characteristics as well as optimization errors that result from the use of relatively noisy daily time-use data. $\tilde{Z}$ as defined in (28) is not a monotonically increasing function of the time inputs, but this is not essential. The efficiency conditions—equality of the marginal productivity and the wage rate—guarantee that optimal time allocations for all observations will be on the upward—sloping part of the production function. Points at which the marginal productivity is negative are therefore unobservable and the production function might equally well be flat on that region. The first order conditions for individuals who are gainfully employed are now:

$$b_s + c_{sm} H_m + c_{sf} H_f = (1 - g_s'(H_s)) W_s \quad s = m, f \tag{30}$$

[13] For convenience we will refer in the following to $\tilde{Z}$ as the production function.

Define $\Sigma_{uu}^{-1} = Q'Q$, where Q is a lower triangular matrix, then

$$\nu \stackrel{\text{def}}{=} Qu|X \sim \mathcal{N}(0, I)$$

Ignoring sample selectivity for the moment, the typical likelihood contribution is given by:

$$\left| \frac{q_{mm}q_{ff}}{\hat{b}_m\hat{b}_f}((c_{mm} + g_m''(H_m)W_m)(c_{ff} + g_f''(H_f)W_f) - c_{fm}^2) \right| \varphi \begin{pmatrix} q_{mm}\hat{u}_m \\ q_{fm}\hat{u}_m + q_{ff}\hat{u}_f \end{pmatrix} \tag{31}$$

where φ is the standard normal density and

$$\left.\begin{array}{l} \hat{u}_s = \log(\hat{b}_s) - x_s'\beta_s \\ \hat{b}_s = (1 - g_s'(H_s))W_s - c_{sm}H_m - c_{sf}H_f \end{array}\right\} s = m, f$$

The jointness functions are specified as in Graham and Green (1984)

$$g_i(H_i) = H_i\left(1 - \frac{1}{1+\delta_i}\left(\frac{H_i}{T}\right)^{\delta_i}\right) \quad i = m, f \tag{32}$$

with $\delta_m, \delta_f \geq 0$. If $\delta_m = \delta_f = 0$, there is no joint production. Increasing δ_m and δ_f raises the amount of joint production, until eventually—at infinitely large values for δ_m and δ_f—all home production time is perceived as leisure. Application of Proposition 1 of the previous section shows that if two jointness functions of this type—with $\delta_i^{(1)}$ and $\delta_i^{(2)}$, say—are observationally equivalent, we must have

$$H_i^{(\delta_i^{(1)} - \delta_i^{(2)})} = \vartheta, \text{ for some } \vartheta \geq 0$$

As a consequence the jointness functions are identified.[14]

The first-order conditions (30) hold for households in which both partners have a job. Until now we have ignored the composition of the sample, but as the sampling rule depends on the labour market status, which is endogenous in this model, the selection process should be given some attention. Considering individuals with the same earnings potential, the ones with low productivity at home will be the most likely to have a paid job. This would imply under–representation of more home-productive individuals in the sample. Consequently the average value of home production would be underestimated. On the other hand, there may be covariates that have a positive (negative) effect on the productivity at home, but an even stronger positive (negative) effect on the probability of being gainfully employed. Other things being equal, the more home productive individuals would be *over–represented* in this case. If the heterogeneity is partially caused by characteristics that are not observed, parameter estimates may be inconsistent. Although it is not possible to state in advance what the direction of the selection bias will be, it is clearly important to account for the selectivity.

[14] As (31) already indicates, this specification of the production function has the additional advantage that it is not necessary to restrict the choice of g_m and g_f—apart from twice differentiability—in order to compute the likelihood contributions analytically. Other specifications lead to similar or less satisfactory empirical results. The identification result allows the inclusion of background characteristics, such as age and schooling, in the jointness functions. In this application, making δ_m and δ_f dependent on schooling and age hardly seemed to improve the estimates. Similar results were found if the elements of C were allowed to depend on age and schooling. We have therefore restricted the role of these 'taste shifters' to the specification of b_m and b_f only.

One way to do this is to solve the complete household-optimization problem. This would lead to a simultaneous model for the participation and home production decisions. The main disadvantage of that approach is that it requires specification of the utility function.[15] Furthermore, modelling the labour market status in this way neglects involuntary unemployment. The selection is not about whether people want a job, but about whether or not wage rates are observed.

The process of sample selection is therefore taken into account by extending the structural model for the home production decisions with a bivariate probit model describing the individuals' employment status.

$$I_s = \begin{cases} 1, & I_s^* \geq 0 \\ 0, & I_s^* < 0 \end{cases} \tag{33}$$

where

$$I_s^* = z_s'\gamma_s + \varepsilon_s \quad s = m, f$$

and

$$\begin{pmatrix} u_m \\ u_f \\ \varepsilon_m \\ \varepsilon_f \end{pmatrix} | x_m, x_f, z_m, z_f \sim \mathcal{N}\left(0, \begin{pmatrix} \Sigma_{uu} & \Sigma_{u\varepsilon} \\ \Sigma_{\varepsilon u} & \Sigma_{\varepsilon\varepsilon} \end{pmatrix}\right)$$

Individual s is employed if and only if $I_s = 1$. The diagonal elements of $\Sigma_{\varepsilon\varepsilon}$ are normalized at 1. In discussing the empirical results the case $\Sigma_{u\varepsilon} = 0$ will be referred to as 'no selectivity'. In the estimates with 'selectivity' the only restrictions on the parameters are those ensuring that the covariance matrix is positive definite. In order to compute the likelihood contributions we use the property:

$$\begin{pmatrix} \varepsilon_m \\ \varepsilon_f \end{pmatrix} | u_m = \overline{u}_m, u_f = \overline{u}_f \sim \mathcal{N}(\Sigma_{\varepsilon u}\Sigma_{uu}^{-1}\overline{u}, \Sigma_{\varepsilon\varepsilon} - \Sigma_{\varepsilon u}\Sigma_{uu}^{-1}\Sigma_{u\varepsilon})$$

For households in which both partners have a paid job, the likelihood contribution is the expression in (31) multiplied by the conditional probability that I_m^* and I_f^* are positive; it is equal to the unconditional probability that at least one of I_m^* and I_f^* is negative otherwise.

4. EMPIRICAL ANALYSIS

4.1. Data

For the empirical application of this model we use-time allocation data from the Swedish HUS–project (Klevmarken and Olovsson, 1986). The 1984 wave of this panel dataset contains information on home production activities. The respondents were asked to report the time they spent on a large number of activities on the day preceding the interview. Sample days were randomly allocated (non-holiday) days over a twelve-month period, with weekend days accounting for nearly half the sample days. We use data on respondents who contributed for two days, one weekday and one weekend day. The time variable for a particular activity is defined as a weighted daily

[15] An attempt was made to estimate a complete model using a linear specification for the marginal rates of substitution between leisure and consumption. In that case it is still possible to derive explicit expressions for the optimal allocations in each of the four possible regimes and for the regime conditions. Satisfactory estimates of the parameters of the utility function were not obtained.

average of the time spent on the weekday and the time spent on the weekend day. This approach implicitly assumes that households solve the maximization problem on a weekly time basis. Note that the model does not address timing constraints (as opposed to time constraints), i.e. it does not describe when during the week a particular activity takes place. The sample we use contains 819 two-adult households and 286 households with one adult. In 517 of the two-adult households both partners were employed and 183 of the 286 heads of single adult households had a paid job.

Home production will here be defined as all activities households reported in the categories *childcare, repair & maintenance* and *household work*. Monetary amounts are expressed in Swedish kronor.[16] An individual's education level is proxied by the total number of years he or she has been enrolled in full time education. For the hourly wages we use the average net wage rate. In the Appendix Table AI summarizes the available information for two-adult households. Summary statistics for single-adult households are in Table AII.

4.2. Estimation Results

The first column of Table I contains the estimates of the model without joint production or selectivity. The parameter estimates of the household production function and the estimates of the bivariate employment-index model are independent of each other. The estimated matrix $\hat{C}$ is negative definite. This restriction was not imposed in estimation. The fact that $\hat{C}$ is negative definite implies that the estimated model satisfies the second order conditions for optimal behaviour that were invariably violated in estimates of Cobb–Douglas and CES models for two-adult households. An explanation for the fact that this essential restriction is satisfied for the quadratic specification can be found in the sign of $\hat{c}_{mf}$, that indicates that male and female home production time are q-substitutes and not q-complements as is implicitly imposed in the Cobb–Douglas specification. Accordingly, the estimates define a well-behaved concave production function.

The presence of young children has a strong positive effect on the home productivity of both males and females. For both partners, the productivity at home increases with age. This may be interpreted as a learning effect, but it may also result from changes in life-style that create a larger potential for home production at higher ages. Of course, this parameter may also capture cohort effects. Other explanatory variables, like familysize and homeownership have a larger and more significant effect on the productivity of females than that of males. The notable exception is the presence of a car, which is predominantly a male affair. The education level of females has a strong negative impact on the productivity at home. Human capital that is relevant for formal jobs appears to be acquired at the cost of domestic skills. As a result, education has a double emancipating effect: not only does it improve a woman's labour market opportunities, it also reduces her comparative advantage in home production.

The parameters of the participation probits are satisfactory. For both partners the probits exhibit highly significant income effects. The participation indices I_s^* depend negatively on the number of young children in the household. As expected the effect is stronger for women than for men. An individual's level of education raises its employment probability as well as that of its partner.

[16] 100 SKr. ≈ £12.87 ≈ \$ 11.12 by the end of 1984.

Table I. Maximum likelihood estimates for two-adult households

	No joint production or selectivity	Joint production and selectivity
Production function		
β_m:		
Constant	4.4469** (6.20)	3.1111** (2.93)
log(age$_m$)	0.2965** (1.99)	0.5451** (2.84)
log(education$_m$)	0.0353 (0.39)	0.0021 (0.02)
dummy homeowner	0.0966 (1.11)	0.0280 (0.32)
log(familysize)	0.1399 (1.27)	0.0478 (0.41)
log(1 + #young children)	0.4610** (4.43)	0.6452** (5.31)
log(1 + #cars)	0.2121** (1.97)	0.1749* (1.79)
β_f:		
Constant	4.8815** (10.2)	4.0181** (4.11)
log(age$_f$)	0.1871** (1.98)	0.4229** (3.41)
log(education$_f$)	−0.2144** (−3.08)	−0.2996** (−3.53)
dummy homeowner	0.1503** (2.52)	0.0766 (1.14)
log(familysize)	0.1823** (2.46)	0.0786 (0.84)
log(1 + #young children)	0.3190** (4.59)	0.5663** (6.25)
log(1 + #cars)	0.0897 (1.24)	0.0680 (0.79)
c_{mm}	−303.56** (−3.44)	−123.80 (−1.43)
c_{mf}	−28.111** (−2.24)	−14.673 (−1.32)
c_{ff}	−70.776** (−3.90)	−42.840 (−1.36)
δ_m		0.1351 (0.70)
δ_f		0.2162 (1.03)
Participation-index male (γ_m):		
Constant	5.9802** (3.13)	5.9961** (2.95)
log(age$_m$)	−2.7447** (−3.89)	−2.6935** (−3.76)
log(education$_m$)	0.4077 (1.54)	0.4209 (1.17)
log(age$_f$)	0.8516 (1.23)	0.7936 (1.01)
log(education$_f$)	0.8306** (2.78)	0.8115** (2.11)
dummy homeowner	0.2492 (1.56)	0.2400 (1.36)
log(familysize)	−0.0160 (−0.06)	−0.0328 (−0.13)
log(1 + #young children)	−0.5026* (−1.85)	−0.5409** (−1.97)
log(1 + #cars)	0.3076 (1.40)	0.3699 (1.56)
Non-labour income	−1.9657** (−10.2)	−1.9747** (−10.5)
Participation-index female (γ_f):		
Constant	2.2099* (1.83)	2.3414** (2.30)
log(age$_m$)	0.2085 (0.39)	0.3288 (0.67)
log(education$_m$)	0.3665** (2.02)	0.3218** (2.08)
log(age$_f$)	−1.1571** (−2.27)	−1.2620** (−2.66)
log(education$_f$)	0.4148** (2.01)	0.4338** (2.46)
dummy homeowner	0.2356* (1.94)	0.1648 (1.44)
log(familysize)	0.2974 (1.65)	0.1856 (1.13)
log(1 + #young children)	−0.9579** (−5.45)	−0.8530** (−5.21)
log(1 + #cars)	0.1396 (0.84)	0.1298 (0.85)
Non-labour income	−0.7049** (−5.33)	−0.6588** (−5.61)
Covariance matrix:		
Var(u_m)	0.5200** (5.87)	0.5619** (6.40)
Cov(u_f, u_m)	0.2053** (4.63)	0.3156** (5.26)
Var(u_f)	0.2361** (9.03)	0.4113** (8.87)
Cov(ε_m, u_m)		0.0175 (0.08)
Cov(ε_m, u_f)		0.1636 (1.48)
Var(ε_m)	1.	1.

(*continued overleaf*)

Table I. (*Continued*)

	No joint production or selectivity	Joint production and selectivity
$\mathrm{Cov}(\varepsilon_f, u_m)$		−0.4814** (−5.78)
$\mathrm{Cov}(\varepsilon_f, u_f)$		−0.6084** (−14.3)
$\mathrm{Cov}(\varepsilon_f, \varepsilon_m)$	−0.0270 (−0.42)	−0.0365 (−0.37)
$\mathrm{Var}(\varepsilon_f)$	1.	1.
Log-likelihood	−2317.9584	−2295.6759
R^2 probit male	0.628	0.629
R^2 probit female	0.278	0.248

t-statistics in parentheses.
* = significant at 10% level.
** = significant at 5% level.

The coefficients of determination for the participation probits are the McKelvey–Zavoina R^2's.[17] The participation probits predict the employment status correctly for 91% of the males and for 77% of the females in the sample. In 71% of the households the predicted employment status for both partners is correct.

In these estimates there is no joint production ($\delta_m = \delta_f = 0$) or selectivity ($\Sigma_{u\varepsilon} = 0$). If there is joint production individuals will be induced to spend more time on home production than they would otherwise do. Ignoring joint production, this phenomenon would incorrectly be attributed to a higher level of marginal productivity. The estimates in the first column of Table I will therefore overestimate the household production function in the presence of joint production. This will in particular affect the estimates of c_{rs} ($r, s = m, f$) and the intercepts in β_m and β_f. If the observed covariates have no effect on the joint production—as is implicitly assumed in the definition of the jointness functions—the other parameters in β_m and β_f will not be systematically biased.

In the previous section it was noted that sample selectivity may lead to over or underestimation of the value of home production. Although the direction of the bias cannot be determined unambiguously one would expect that—overall—home productive individuals will be underrepresented in the subsample of households in which both partners are employed. With respect to unobserved heterogeneity this corresponds to a negative correlation between u_s and ε_s ($s = m, f$). Considering observed covariates that affect home productivity, but hardly affect the earnings capacity, we would then expect the parameter estimates to be biased towards zero.[18] More specifically this is expected to hold for the number of young children. The presence of a young child extends the

[17] They are defined as $R^2 = D/(D + N)$, where

$$D = \sum_{i=1}^{N} (z_{is}'\hat{\gamma}_s)^2 - \frac{1}{N}\left(\sum_{i=1}^{N} z_{is}'\hat{\gamma}_s\right)^2 \qquad s = m, f$$

The McKelvey–Zavoina R^2 approximates the R^2 of the latent index equation (33). It measures the variability of $z_{is}'\hat{\gamma}_s$ relative to the normalized effect of the random errors. It does, however, not measure the goodness of fit of the probit equation. To this end one could compute the R^2 of the homoscedastic regression representation $I_{is}/\sqrt{p_{is}(1 - p_{is})} = \sqrt{p_{is}/(1 - p_{is})} + e_{is}$, $i = 1, \ldots, N$, $s = m, f$. This R^2 is 0.981 for the males and 0.481 for the females. See Veall and Zimmermann (1996) for a comparison of alternative pseudo-R^2's.

[18] Put differently, covariates that have opposite effects on the productivity and the participation probability will appear to have a smaller effect on productivity. The reverse holds for variables that affect productivity and participation probability in the same direction.

opportunities for home production and the marginal home productivity will be raised. The wage rate will not be affected and—as the separate estimates of the participation probit indicate—both partners will have a lower propensity to work in a paid job. From this it follows that the effect of the presence of young children on home production is underestimated. Households for which an additional young child has a relatively low impact on the productivity at home (e.g. parents living in the same house, easy access to and/or less moral objections against child care facilities) will have a larger probability of being in the sample. This way, the effect of the number of young children on home production will be flattened. Conversely, the number of cars has a positive effect on both the employment probability and on the productivity. This parameter will by a similar reasoning be overestimated.

The second column of Table I contains the parameter estimates for the model with joint production and selectivity. The first thing to notice is that the covariance between ε_f and u_f is large and negative; the correlation coefficient is -0.95. The correlation between ε_f and u_m is -0.64. The correlation between ε_m and u are much smaller in absolute value. The sample selection effect runs mainly through the participation equation of the female. This is in accordance with the familiar observation that female labour supply is far more responsive to changes than male labour supply. The predicted under and overestimations that would result from ignoring sample selection are confirmed. The estimated β-parameters for the number of young children in the household increase by 40% for males and 80% for females. The double emancipating effect of the woman's education is also more prominent than in the estimates in the first column. These changes are entirely due to incorporating selectivity. Introduction of joint production—before or after sample selectivity is included—leave the β parameters virtually unchanged, only reducing the constant term. The estimates of the c_{rs} parameters are more than halved by the introduction of joint production. Conversely, the introduction of correlations between ε and u has a strong effect on the β parameters, but hardly affects the estimates of the elements of C. Table II summarizes the likelihood ratio tests regarding joint production and sample selectivity.

Clearly there is strong evidence of selectivity. The evidence of joint production is somewhat weaker, but more corroborative than would be expected from the t-values of the estimates of δ_m and δ_f. Joint production appears to be more important for women than for men. Although $\hat{c}_{mf}$ is not significant, the LR test of $c_{mf} = 0$ firmly supports the hypothesis that male and female home production time are q-substitutes. The LR tests also reject the hypothesis $c_{mm} = c_{ff}$ although the parameter estimates are not significant (at 10%).

Table II. Likelihood ratio tests for two-adult households

Test	Maintained hypothesis	Value	d.f.	5% CV
Selectivity and jointness		44.57**	6	12.6
Selectivity	No jointness	38.81**	4	9.49
Selectivity	Jointness	37.68**	4	9.49
Jointness	No selectivity	6.88**	2	5.99
Jointness	Selectivity	5.75*	2	5.99
$c_{mf} = 0$	Selectivity and jointness	17.14**	1	3.84
$c_{mm} = c_{ff}$	Selectivity and jointness	4.62**	1	3.84

* = significant at 10% level.
** = significant at 5% level.

The latter inference highlights a result that was found in all specifications that were estimated. The C parameters determine the slope of the marginal home productivity schedules. The asymmetry between the rate of decline of the marginal productivity at home for males and females may reflect the fact that the allocation of time is less elastic for males than for females. This is a well-known property for labour supply and would appear to extend to home production decisions. It must be noted that the interpretation of the relative magnitudes of c_{mm} and c_{ff} is not that straightforward. Holding all other parameters constant, a more negative value of c_{ss} corresponds to a uniformly lower marginal productivity schedule for individual s. Optimal home production time for individual s will therefore be reduced. Both effects lead to a lower level of output from home production, but the effect on the average level of home production per hour is ambiguous.[19] Keeping the (observed) optimal values of H_m and H_f fixed, on the other hand, a higher value of $|c_{ss}|$ corresponds to a higher value of b_s and a MP schedule which is higher on the interval $[0, H_s]$. Total home production and average production per hour will be higher. In order to interpret the differences between c_{mm} and c_{ff} or between different estimates of C, we have to consider the differences in the other parameters simultaneously.

Table III presents sample averages of estimates of some key variables characterizing the home production technology and decisions. From the sample averages of $\hat{b}_m$ and $\hat{b}_f$ it can be seen that the marginal productivity at home is initially higher for males, but falls more rapidly than the marginal productivity of females. The range of values of H for which the marginal home productivity of females is positive is more than twice as large as the corresponding range for males. Roughly speaking, we may conclude that the marginal productivity of males is higher over the first hour of home production, but falls below the marginal productivity of females after that. A possible explanation for this phenomenon is that men have specialized in activities for which market alternatives are more expensive.

Table III also reports the sample average of the estimated value of one hour of home production. Allowing for joint production and sample selection practically halves this value. The estimated

Table III. Sample averages (and standard deviations)

	No joint production or selectivity	Joint production and selectivity
$\hat{b}_m$	473.27 (97.48)	250.62 (59.57)
$\hat{b}_f$	257.19 (42.99)	183.44 (39.77)
$\tilde{Z}/(H_m + H_f)$	239.09 (141.52)	122.81 (63.47)
$g_m(H_m)/H_m$		0.411 (0.086)
$g_f(H_f)/H_f$		0.477 (0.079)
P.E.S.**	0.291 (0.558)	0.320 (0.407)
P.E.C.**	−48.44 (74.64)	−28.13 (38.04)
El. of Scale	0.172 (0.108)	0.198 (0.095)
$\varepsilon(H_m, W_m)$***	−0.284 (0.705)	−0.296 (0.460)
$\varepsilon(H_m, W_f)$	0.095 (0.221)	0.096 (0.184)
$\varepsilon(H_f, W_m)$	0.026 (0.040)	0.031 (0.041)
$\varepsilon(H_f, W_f)$	−0.269 (0.627)	−0.203 (0.197)

* P.E.S. = partial elasticity of substitution.
** P.E.C. = partial elasticity of complementarity.
*** $\varepsilon(X, Y)$ is the elasticity if X w.r.t. Y.

[19] For the case of single-adult households without joint production, the two effects cancel and the average value of home production is independent of c.

value is nevertheless higher than the estimates presented in Gronau (1980). Gronau used the 1973 data of the Michigan Study of Income Dynamics, but only considered the home production of the female. On the other hand, our sample average of SKr. 122.81 lies well within the range of values that Homan, *et al.* (1987) computed for Dutch time-allocation data. While accounting for jointness and endogenous sampling reduces the estimated monetary equivalent of home production time by 50%, the average values of $g_m(H_m)/H_m$ and $g_f(H_f)/H_f$ indicate that between 40% and 50% of home production time is valued as leisure, more so for women than for men. One should, however, note that these figures are point estimates based on relatively imprecise parameter estimates.

The partial elasticity of substitution (P.E.S.) in Table III is defined as the relative change in H_m/H_f in response to a small change in W_m/W_f. Defined in this way, the elasticity incorporates the effect of joint production:

$$\begin{aligned}\text{P.E.S.} &= \frac{\mathrm{d}\log(H_m/H_f)}{\mathrm{d}\log(W_m/W_f)} \quad \text{s.t.} \quad \mathrm{d}F(H_m, H_f) = 0 \\ &= -\frac{W_m(1-g'_m)W_f(1-g'_f)(W_m H_m(1-g'_m) + W_f H_f(1-g'_f))}{H_m H_f (b+CH)'\Lambda(b+CH)}\end{aligned}$$

with

$$\Lambda = \begin{pmatrix} c_{ff} + W_{fg''_f} & -c_{mf} \\ -c_{mf} & c_{mm} + W_{mg''_m} \end{pmatrix}$$

This elasticity reflects the ease with which H_m and H_f can be substituted for each other following a small change in their relative prices. In a production technology with only two factors of production the factors are always p-substitutes. With regard to the complementarity of H_m and H_f it is thus more interesting to investigate how the marginal product of one factor of production changes when more of the other factor is used. This concept is formalized by the partial elasticity of complementarity (P.E.C.). Following Sato and Koizumi (1973) we define:

$$\begin{aligned}\text{P.E.C.} &= \frac{\mathrm{d}\log(\partial\tilde{Z}/\partial H_m)}{\mathrm{d}\log\tilde{Z}} \quad \text{s.t.} \quad \mathrm{d}H_m = 0 \\ &= \frac{c_{mf}\tilde{Z}}{(b_m + c_{mm}H_m + c_{mf}H_f)(b_f + c_{mf}H_m + c_{ff}H_f)}\end{aligned}$$

A negative elasticity of complementarity implies that an increase in one factor of production, reduces the marginal product of the other. At constant prices less of that factor will be used. In this sense the factors of production behave as substitutes for each other, so called q-substitutes. Clearly, H_m and H_f are q-substitutes.

The elasticity of scale is a local measure of the returns to scale:

$$\begin{aligned}\text{El. of scale} &= \frac{\partial\tilde{Z}}{\partial H_m} + \frac{\partial\tilde{Z}}{\partial H_f} \\ &= \frac{b'H + H'CH}{b'H + \frac{1}{2}H'CH}\end{aligned}$$

The wage elasticities presented in Table III refute the claim that—as in labour supply—males are less elastic in their allocation of time to home production than females. In fact the wage elasticities of H_m are slightly higher than those of H_f.

Figures 1 and 2 contain the isoquants corresponding to the estimates of the model with joint production and sample selection. These are drawn for a household in which both partners are 30 years of age and have 12 years of education. Figure 1 refers to a household without children; in Figure 2 the household has one young child. The expansion path for $W_m = W_f$ intersects with the H_m axis, but at the relevant wage rates H_f is greater than H_m. The presence of a young child in the household shifts the production function outward and increases the bliss-level by a factor 2.5.

4.3. Simulations

With the estimated structural parameters home production decisions can be computed for a specific household, provided both partners are gainfully employed. To investigate how home production decisions are affected by the exogenous variables, the mean and standard deviation of H_m, H_f and the average value of home production per hour are computed for some household types of interest. These values were computed on the basis of 10 000 random draws from the four-dimensional normal probability distribution with the estimated covariance matrix from the model with joint production and sample selection. Only the draws that satisfied both participation conditions were kept. For the remaining households the optimum values for H_m and H_f were solved numerically. The results are summarized in Table IV. The reference household type has two partners of age 30, with 12 years of education, an average net wage rate of SKr. 35.00, no children, no cars and they do not own their house. Inasmuch as the decisions can only be computed for households in which both partners are gainfully employed, the numbers in the table refer to conditional means and standard deviations. The selection probability (*part. prob.*) denotes the share of households of the specified type to which the means and standard deviations apply.

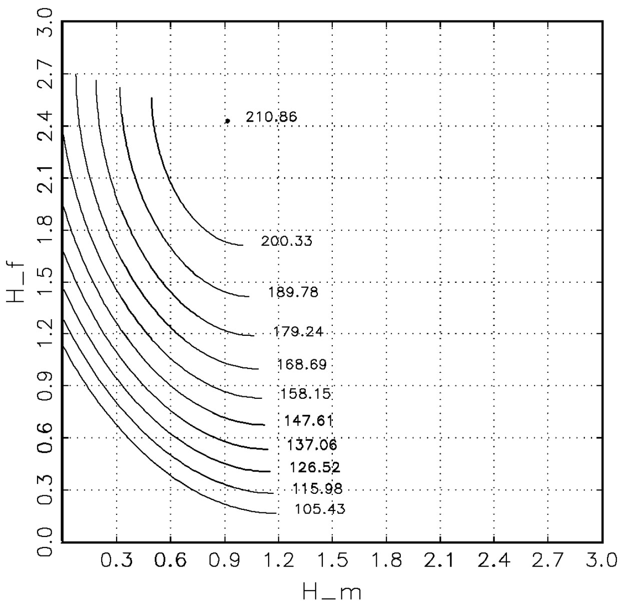

Figure 1. Contour lines of the home production function for a household without children (both partners aged 30, 12 years of education, no homeowners, no car, median w.r.t. unmeasured variation)

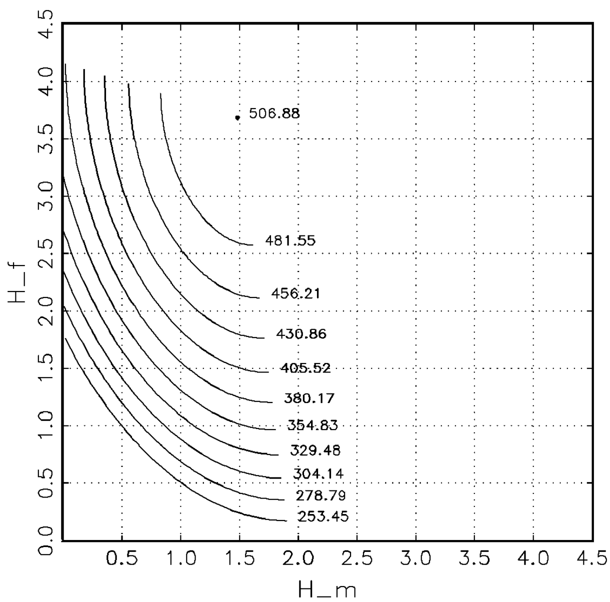

Figure 2. Contour lines of the home production function for a household with one young child (both partners aged 30, 12 years of education, no homeowners, no car, median w.r.t. unmeasured variation)

Age has a pronounced effect on home production. H_m, H_f as well as the value of home production per hour grow with age. At higher ages, the age effect is slightly flattened by the logarithmic specification.[20] Remarkably, the effect of age on H_m is about twice as large as on H_f. If the effect of age would be a pure cohort effect one would expect the opposite to hold. Raising the age of both partners from 20 to 60 raises H_m by 64%, H_f by 23% and the value of one hour of home production by 35%. Even then, H_m at 60 is lower than H_f at age 20. As the selection probability falls with age, the age effect may be expected to be even larger in the population as a whole.

Raising the wages of both partners reduces the time spent on home production, but increases the average value of home production. Raising the wage rate of one partner, reduces that person's home production time. This reduction in home production time is only partially replaced by his or her partner. An increase of the female's average net wage from SKr. 30 to SKr. 50, holding the male's wage rate fixed at SKr. 30, reduces H_f by 12%, while H_m is increased by only 2.8%. The total value of home production falls by 2%. Notice that the sample selection equations are reduced form equations in the sense that they do not depend on the wage rates. The selection probability in the table is therefore insensitive to changes of the wage rates.

The level of education hardly affects male productivity at home, but has a significant negative impact on the home productivity of the female. The effect of a higher level of education on the mean values in Table IV consists of a pure education effect as well as a composition effect. Considered

[20] The model has also been estimated with b_m and b_f specified as a quadratic function of age and log age. These specifications showed a similar monotonous increase of home production with age. The likelihood values were only slightly higher, but most parameters became considerably less significant. The reason for this is that the number of parameters of the model is already large and the proportion of individuals with age over 60 was small.

Table IV. Simulation results for various household types

Age					
male	20	30	40	50	60
female	20	30	40	50	60
part. prob.	0.937	0.867	0.780	0.681	0.579
H_m	0.791	0.972	1.092	1.202	1.298
	(0.83)	(0.97)	(1.05)	(1.15)	(1.25)
H_f	1.827	2.005	2.122	2.205	2.293
	(1.25)	(1.25)	(1.23)	(1.19)	(1.19)
$\tilde{Z}/(H_m + H_f)$	77.79	87.45	94.00	99.71	104.93
	(44.20)	(49.43)	(53.36)	(57.36)	(62.76)
Wage rates					
male	30	40	50	50	30
female	30	40	50	30	50
part. prob.	0.867	0.867	0.867	0.867	0.867
H_m	0.989	0.955	0.921	0.894	1.017
	(0.97)	(0.97)	(0.96)	(0.96)	(0.97)
H_f	2.062	1.949	1.841	2.093	1.811
	(1.26)	(1.24)	(1.21)	(1.26)	(1.21)
$\tilde{Z}/(H_m + H_f)$	85.83	89.06	92.23	87.02	90.59
	(49.12)	(49.75)	(50.42)	(49.63)	(49.98)
Years of schooling					
male	6	12	6	12	18
female	6	6	12	12	18
part. prob.	0.681	0.770	0.806	0.867	0.926
H_m	0.797	0.831	0.919	0.972	1.056
	(0.84)	(0.87)	(0.91)	(0.97)	(1.04)
H_f	2.253	2.418	1.882	2.005	1.828
	(1.17)	(1.33)	(1.11)	(1.25)	(1.25)
$\tilde{Z}/(H_m + H_f)$	85.44	89.56	83.46	87.45	88.39
	(41.84)	(44.31)	(46.18)	(49.43)	(53.85)
Presence of (young) children					
# children	0	1	2	2	2
# young children	0	1	2	1	0
part. prob.	0.867	0.712	0.594	0.724	0.885
H_m	0.972	1.442	1.796	1.467	1.005
	(0.97)	(1.37)	(1.67)	(1.39)	(1.01)
H_f	2.005	2.650	3.096	2.757	2.169
	(1.25)	(1.40)	(1.52)	(1.48)	(1.37)
$\tilde{Z}/(H_m + H_f)$	87.45	115.95	136.87	118.50	91.40
	(49.43)	(67.66)	(82.81)	(68.51)	(51.61)

Miscellaneous	Ref.	House-owners	1 car
part. prob.	0.867	0.902	0.889
H_m	0.972	1.012	1.151
	(0.97)	(1.02)	(1.12)
H_f	2.005	2.309	2.131
	(1.25)	(1.47)	(1.37)
$\tilde{Z}/(H_m + H_f)$	87.45	93.95	96.66
	(49.43)	(52.60)	(57.28)

Note: In the reference household both partners are 30 years of age, have 12 years of education, net marginal wage rates of 35 SKr., no children, no car, and they do not own the house they live in. H_m and H_f are measured in hours per day. Standard deviations in parentheses.

in isolation, the male education level has hardly any effect. The changes of the conditional means in Table IV mainly reflect the composition effect: households in which the man has a relatively high productivity at home now enter the subsample of working couples, raising the average value of H_m, H_f and average home production. For the education level of females, this composition effect is offset by the fall in the productivity at home. H_f is reduced and the average value of home production falls as well. Notice that a simultaneous increase of the level of education of males and females leads to a considerable shift of home production time from the female to the male.

The presence of young children in the household obviously has a very large effect on home production. The birth of one young child raises the total value of home production by 82%, whereas H_m and H_f increase by 48% and 32% respectively. The birth of a twin increases the value of home production by 157%. The presence of two children of age over 7 increases the probability that both parents have a paid job and raises H_m and H_f by 3% and 8% respectively. The total value of home production is raised by 11%.

The effect of owning a car on the value of home production is larger than the effect of homeownership. This may be due to the fact that the presence of a car not only increases the opportunities for home production—predominantly a male task though—but also raises the productivity of home production time for both partners. Owning the house one lives in raises the total value of home production by 20%. The presence of a car increases the total value of home production by 22%.

4.4. Specification Checks

Although the LR-tests in Table II are favourable to the estimates in the second column of Table I, it is important to check the validity of the complete set of assumptions underlying the model.[21] The residuals $\hat{u}$ can only be computed for the subsample of 519 households in which both partners are employed. Conditional on selection, the random terms u are non-normal. For a direct test of the normality assumption, we would have to generalize the four-dimensional normal probability distribution and integrate over the relevant areas. Instead, we make use of the generalized residuals:

$$\eta_{mi} = \Pr(u \leq u_{mi} | x_{mi}, z_{mi}, z_{fi}, \varepsilon_m > -z'_{mi}\gamma_m, \varepsilon_f > -z'_{fi}\gamma_f; \theta) \tag{34}$$

where θ is shorthand for the model parameters. If the assumptions about the functional forms and the probability distribution of u and ε are correct, η_{mi} has a uniform distribution on [0,1]. For females, η_{fi} is defined analogously. Replacing the unknown parameters by their estimates from the second column of Table I and using the normality assumption, we can compute $\hat{\eta}_{mi}$ and $\hat{\eta}_{fi}$ for each observation by means of simulation. As an informal specification test we check whether the empirical distribution of $\hat{\eta}_m$ and $\hat{\eta}_f$ are sufficiently close to the 45° line. The results are in Figures 3 and 4. The maximum vertical distance between the empirical distribution function and the 45° line is the Kolmogorov–Smirnov test statistic. If the true parameter values are used, this is a well-defined test statistic for which the small-sample probability distribution is known.[22] The 5% critical value of the Kolmogorov–Smirnov test has been drawn as an upper and a lower band around the 45° line. For males as well as for females, the estimated empirical distribution function

[21] The numerical instability of the information matrix prohibits the use of Hausman-type tests.
[22] Using the parameter estimates instead, the small sample properties of the Kolmogorov–Smirnov test no longer hold because of the combined effect of parameter uncertainty and overfitting (see e.g. Cox and Hinkley, 1974).

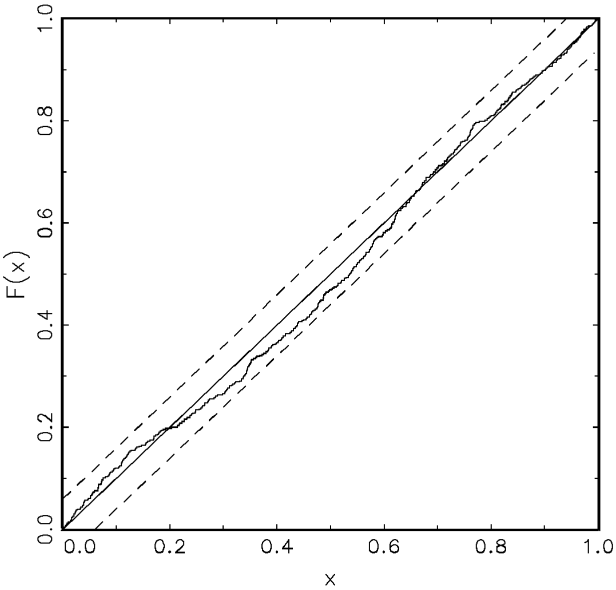

Figure 3. Empirical distribution function of the generalized residuals for males

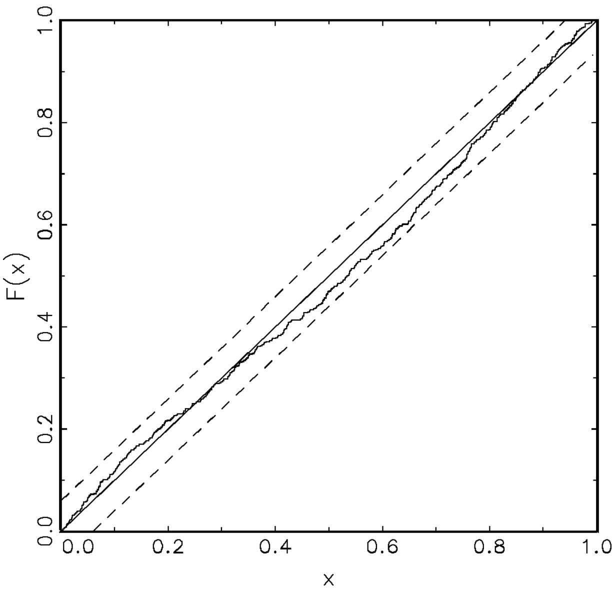

Figure 4. Empirical distribution function of the generalized residuals for females

stays within these bands. The maximum vertical distance between the curves is 0.0458 for males and 0.0442 for females. The 5% critical value for the Kolmogorov–Smirnov test is 0.0598 for a sample size of 517. It is interesting to note that the estimated empirical distribution function for

Table V. Results of the conditional moment tests

	d.f.	5% C.V.	Males	Females
First moment	1	3.84	0.51	0.37
First two moments	2	5.99	2.16	0.40
Quartiles	3	7.81	3.61	4.74
Quintiles	4	9.49	9.54*	5.85
Deciles	9	16.90	12.20	17.98*
Quartiles and first 2 moments	5	11.10	6.33	7.63
Quintiles and first 2 moments	6	12.60	11.01	7.41
Deciles and first 2 moments	11	19.70	13.13	19.86*

* = rejects at 5% level.
** = rejects at 1% level.

the estimates in the first column of Table I (no joint production, no sample selection) fails to stay within the bands. For those estimates, the value of the Kolmogorov–Smirnov test is 0.0632 for males and 0.0767 for females.

The generalized residuals (34) can also be used as the basis for a formal (asymptotic) specification test. The property that η_{mi} and η_{fi} follow a uniform distribution on [0, 1] can be inspected with a conditional moment test (CM test) along the lines of Newey (1985). From the distribution of η_{mi} and η_{fi} one derives conditional moment restrictions that are true under the null hypothesis of a correctly specified model. For each observation, the value of these restrictions is computed and appended to the vector of scores (the moment restrictions that were used to estimate the model). The test statistic can now easily be computed as $N \times \mathrm{R}^2$, where R^2 is the coefficient of determination of the OLS regression of a vector of ones on the vector described above. This test statistic has an asymptotic χ^2 distribution with its degrees of freedom equal to the number of moment restrictions. As moment restrictions we use non–central moments of η_s: $\mathrm{E}(\eta_s^k|x_m, x_f, z_m, z_f) = 1/(k+1)$, and quantiles of the distribution of η_s: $\mathrm{E}(1_{[\eta_s<\alpha]}|x_m, x_f, z_m, z_f) = \alpha$. Each choice of the restrictions generates a different CM test focusing on different aspects of the distributional assumption. Compared to the graphical test the CM test will only test for a subset of the infinitely many restrictions characterizing a probability distribution. On the other hand, the CM test takes account of the conditional nature of the moment restrictions whereas the graphical test only looks at the marginal distribution of η_s.

Table V summarizes the values of the test statistic for several combinations of conditional moment restrictions. For males the test statistic based on quintiles lies just above the 5% critical value. Surprisingly, moving on to deciles the problem disappears but now the test–statistic for females is slightly too high. This is still the case if deciles are combined with the first two moments of η_f. None of the other tests rejects at the 5% level.[23]

4.5. Empirical Results for the Single-adult Case

In order to gain some further insights into the model and the estimates discussed above, the model is also estimated for single-adult households, even though the number of observations is too small

[23] The CM test is a formal asymptotic test, but its behaviour in small samples is unknown and is approximated by the asymptotic properties. According to Davidson and MacKinnon (1993) these tests have poor finite–sample properties and tend to reject the null hypothesis too often when it is true. In the light of this observation the results in Table V reinforce the belief that the model describes the data adequately.

Table VI. Maximum likelihood estimates for single-adult households

	Males	Females
Production function β:		
Constant	3.3141* (1.88)	4.2497** (3.42)
log(age)	0.4741 (1.45)	0.0864 (0.38)
log(education)	0.1852 (0.51)	−0.0907 (−0.29)
dummy homeowner	0.2228 (1.05)	0.3194** (1.85)
log(familysize)	0.3556 (1.34)	0.4204** (2.69)
log(1 + # young children)	0.6165 (1.09)	0.7418* (1.83)
log(1 + # cars)	0.0308 (0.12)	0.1484 (0.72)
c	−235.12 (−1.27)	−75.16* (−1.82)
Participation index (γ):		
Constant	−71.558** (−3.99)	−43.222** (−3.10)
log(age)	36.477** (3.77)	25.435** (3.27)
$(\log(\text{age}))^2$	−4.9081** (−3.73)	−3.6199** (−3.41)
log(education)	2.5717** (2.70)	0.0860 (0.20)
dummy homeowner	−0.2661 (−0.68)	0.0951 (0.31)
log(familysize)	0.0939 (0.18)	0.6334** (2.33)
log(1 + # young children)	−0.5951 (−0.50)	−1.0565 (−1.44)
log(1 + # cars)	0.4293 (0.85)	0.5405 (1.36)
Non-labour income	−2.8238** (−4.08)	−3.1313** (−5.93)
σ_{uu}	0.6669** (3.42)	0.5416** (3.26)
$\sigma_{u\varepsilon}$	−0.3072 (−1.16)	0.6827** (5.67)
Log-likelihood	−148.6411	−213.6639
R^2 employment probit	0.723	0.666
Sample averages (and standard deviations):		
$\hat{b}$	311.44 (114.16)	120.39 (48.19)
$\tilde{Z}/H$	208.47 (162.03)	105.48 (65.21)
Elasticity of scale	0.243 (0.20)	0.369 (0.22)
$\varepsilon(H, W)$	−0.266 (0.44)	−0.826 (3.22)

Note: *t*-statistics in parentheses for the estimates; standard deviations for the sample averages.
* = significant at 10% level.
** = significant at 5% level.

to expect high accuracy. Table VI contains the parameter estimates. The model is estimated for males and females separately. Since joint production cannot be identified in a sufficiently flexible model for households with only one adult (see Section 2), we only take sample selection into account. In our sample we have 122 households with one male adult and 164 with one female adult. In respectively 83 and 100 of these households the adult had a paid job.

Most parameter estimates are not significant, though the signs and magnitudes are all reasonable. Most importantly, the estimates of c are in both cases negative. Again, $\hat{c}$ is much more negative for males than for females. Taken together with the average values of $\hat{b}$, this corresponds to initially higher but sharper declining MP schedules for males than for females. Again the marginal product is initially higher for males, but falls below that of females after the first hour. For females, the average estimate of the value of home production per hour over the sample is lower than for the two-adult case, but this is by and large caused by differences in family composition. The difference between the value of home production for males and for females is considerable, but it must be noted that estimates of the one-adult model on pooled data show that the differences between parameter estimates for males and females are not significant. (Notice that the tests in Table II indicate that in the model for two-adult households the values of c_{mm} and c_{ff} are significantly different.) The small number of one-adult households makes it difficult to draw sharp conclusions as to whether the level of output of home production in single adult households is gender-dependent or not. Nevertheless, the two sets of estimates are very different with respect to selectivity. For the males the relevant parameter, $\sigma_{u\varepsilon}$, is small and not significant, but for females it is positive and significant. This indicates that unobserved heterogeneity has the same effect on the productivity at home as on the probability of being employed. The McKelvey–Zavoina R^2 of the employment probit is again smaller for females than for males. The percentage of households for which the employment status is predicted correctly on the basis of these estimates equals 87% for both sets of estimates.

Figures 5 and 6 present the graphical specification tests for the one-adult models. For both sets of estimates the empirical distribution function of the generalized residual η (cf. (34)) stays close to the 45° line. The value of the Kolmogorov–Smirnov test statistic is 0.0542 for males and 0.0643 for females. The 5% critical value of the Kolmogorov–Smirnov test is 0.1492 for a sample size of 83 and 0.136 for a sample size of 100.

Although the estimates of the two-adult model are more significant, the parameters that have the largest impact on the computation of output levels—the elements of C, representing the returns to scale—are relatively weak. Ignoring joint production we can rewrite the first-order condition (30) as:

$$H = b^0 + C^{-1}W \tag{35}$$

where $b^0 \stackrel{\text{def}}{=} -C^{-1}b$ is the *bliss point* of $\tilde{Z}$. From this it is clear that C represents the effect wages have on home production time. Large negative values for the elements of C reflect a weak response of home production time to variation in wage rates (i.e. a weak observed correlation between H

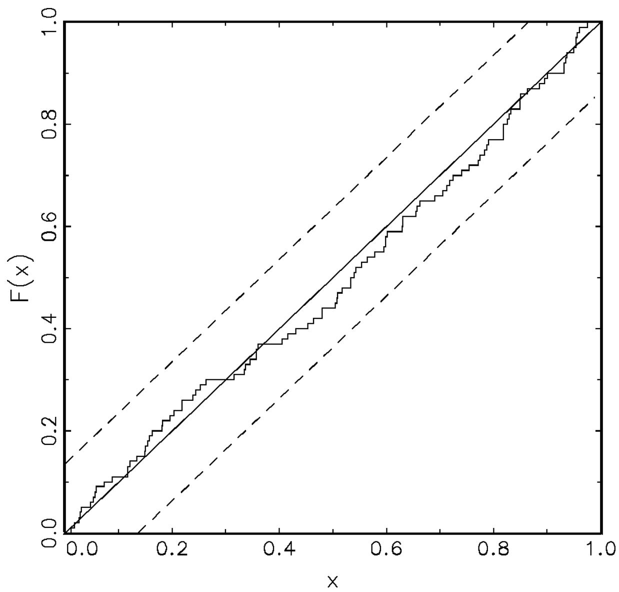

Figure 5. Empirical distribution function of the generalized residuals for single males

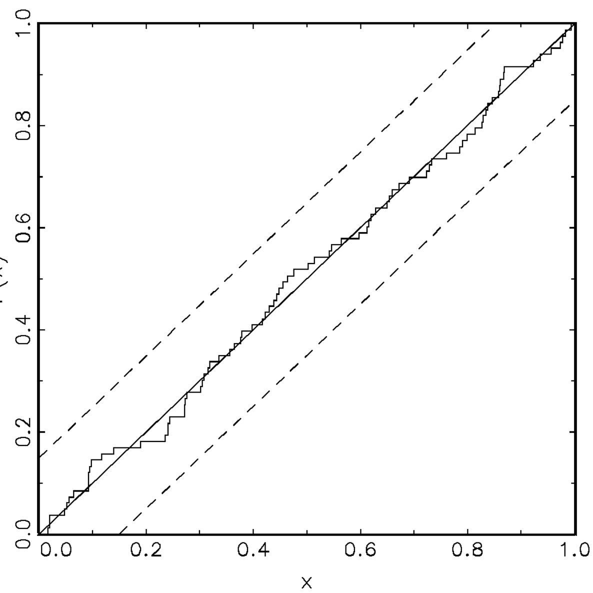

Figure 6. Empirical distribution function of the generalized residuals for single females

and W). This will to some extent be caused by the overdispersion inherent in measuring daily time-allocations as compared to the optimal allocations. The correlation may also be affected by the tax system. As the parameters of the household production function are estimated on the information from the subset of 517 working couples only, it seems promising to incorporate the information of the remaining 302 households as well. In order to do so we have to solve and estimate the complete model. This requires specification of the utility function (see footnote 15).

5. CONCLUDING REMARKS

The analysis illustrates that a dichotomous household production model extended with the possibility of joint production provides a useful framework for analyzing time allocation data. The dichotomy property for the decisions in households in which both partners have a job, simplifies an otherwise complicated analysis considerably. It was shown, that in a model for two-adult households, it is possible to identify joint production non-parametrically on the basis of time-allocation data only. For one-adult households that property is lost, implying that the analysis is in fact restricted to the Gronau model without joint production.

The estimation results in this paper satisfy the regularity conditions of the behavioural model, and specification tests show no conflict between the data and the estimated model. The present results suggest that the highly unsatisfactory parameter estimates found in earlier studies stem from the fact that time inputs were restricted to be q-complements. For the data used in this paper the time inputs are clearly shown to be q-substitutes instead. A remarkable phenomenon in these estimates is that returns to scale are more important for men than for women. A man's marginal productivity at home is initially higher, but declines at a faster rate and falls below that of his partner if home production time is one hour or more. Among the variables that have a strong effect on home production are the ages of the adults in the household, the number of young children and the female's level of education. More education diminishes her marginal home productivity. The estimates also indicate that sample selection is important. However, taking account of sample selectivity does not affect the estimates of the returns to scale or the level of output notably. Joint production is more important for women than for men, but its effect is small in both cases.

APPENDIX

Proof of Proposition 1

For notational convenience the arguments H_m and H_f will be suppressed. We will abbreviate expressions like

$$\frac{\partial^2 z}{\partial H_m \partial H_f}(H_m, H_f, \psi_Z(H_m, H_f)) \quad \text{as} \quad \frac{\partial^2 Z}{\partial H_m \partial H_f}(\psi_Z)$$

Differentiating (17) with respect to H_f and (18) with respect to H_m gives

$$\frac{\partial^2 Z}{\partial H_m \partial H_f}(\psi_Z) + \frac{\partial^2 Z}{\partial H_m \partial X_Z}(\psi_Z)\frac{\partial \psi_Z}{\partial H_f} = \frac{1-g'_m}{1-k'_m}\left(\frac{\partial^2 F}{\partial H_m \partial H_f}(\psi_F) + \frac{\partial^2 F}{\partial H_m \partial X_Z}(\psi_F)\frac{\partial \psi_F}{\partial H_f}\right) \tag{A1}$$

$$\frac{\partial^2 Z}{\partial H_f \partial H_m}(\psi_Z) + \frac{\partial^2 Z}{\partial H_f \partial X_Z}(\psi_Z)\frac{\partial \psi_Z}{\partial H_m} = \frac{1-g'_f}{1-k'_f}\left(\frac{\partial^2 F}{\partial H_f \partial H_m}(\psi_F) + \frac{\partial^2 F}{\partial H_f \partial X_Z}(\psi_F)\frac{\partial \psi_F}{\partial H_m}\right) \tag{A2}$$

for $(H_m, H_f) \in \breve{\mathcal{H}}$. From the definition of ψ_Z and ψ_F we have

$$\frac{\partial Z}{\partial X_Z}(\psi_Z) = 1 = \frac{\partial F}{\partial X_Z}(\psi_F) \tag{A3}$$

on $\mathcal{H}$. Differentiating these two equalities with respect to H_m and H_f, respectively, gives for $(H_m, H_f) \in \breve{\mathcal{H}}$

$$\frac{\partial^2 Z}{\partial X_Z \partial H_i}(\psi_Z) + \frac{\partial^2 Z}{(\partial X_Z)^2}(\psi_Z)\frac{\partial \psi_Z}{\partial H_i} = 0, \quad i = m, f$$

$$\frac{\partial^2 F}{\partial X_Z \partial H_i}(\psi_F) + \frac{\partial^2 F}{(\partial X_Z)^2}(\psi_F)\frac{\partial \psi_F}{\partial H_i} = 0, \quad i = m, f$$

These equations imply that the left-hand sides of (A1) and (A2) are equal on $\breve{\mathcal{H}}$. The same goes for the terms in parentheses on the right-hand sides of (A1) and (A2). By assumption (20), this implies that for almost every $(H_m, H_f) \in \breve{\mathcal{H}}$:

$$\frac{1-g'_m(H_m)}{1-k'_m(H_m)} = \frac{1-g'_f(H_f)}{1-k'_f(H_f)} = \vartheta \tag{A4}$$

for some non-negative constant ϑ. By the continuity of the-first order derivatives of the jointness functions this property holds for every $(H_m, H_f) \in \check{\mathcal{H}}$. QED.

Proof of Proposition 2

Under the assumptions we made about g, we have that for $H \in [H_0, H^*]$:

$$0 \leq \frac{g'(H)}{1-g'(H)}\frac{\partial Z}{\partial H}(H, \psi_Z(H)) \leq \frac{1}{1-g'(H)}\frac{\partial Z}{\partial H}(H, \psi_Z(H)) \tag{A5}$$

The term on the right is equal to the wage rate at which $(H, \psi_Z(H))$ is an optimal choice. Therefore, the integrand is bounded on $\mathcal{H}$ and the integral exists. Having established that definition (27) is sound, we have to show that this function satisfies (25) and (26) and is admissible. It is straightforward to verify that F satisfies (25). Equality of ψ_Z and ψ_F on $\mathcal{H}$ follows directly from:

$$\frac{\partial Z}{\partial X_Z}(H, \psi_Z(H)) = 1 = \frac{\partial E}{\partial X_Z}(H, \psi_F(H)) \tag{A6}$$

Clearly, F is twice differentiable and $\partial F/\partial X_Z$ is continuously differentiable. By the first inequality in (A5), F is also strictly increasing. The household production decisions for a one-adult household, where the adult has a paid job, can be described by the following optimization problem:

$$\max_{H,X_Z} Z(H, X_Z) + W_g(H) - X_Z - WH \tag{A7}$$

By assumption, this problem has a unique interior solution for each $W \in \mathcal{W}$. Therefore the following second-order conditions have to hold:

$$\frac{\partial^2 Z}{(\partial X_Z)^2}(\hat{H}, \psi_Z(\hat{H})) < 0 \tag{A8}$$

$$\frac{\partial^2 Z}{(\partial H)^2}(\hat{H}, \psi_Z(\hat{H})) + g''(\hat{H})W < 0 \tag{A9}$$

$$\frac{\partial^2 Z}{(\partial X_Z)^2}(\hat{H}, \psi_Z(\hat{H}))\left\{\frac{\partial^2 Z}{(\partial H)^2}(\hat{H}, \psi_Z(\hat{H})) + g''(\hat{H})W\right\} - \left[\frac{\partial^2 Z}{\partial X_Z \partial H}(\hat{H}, \psi_Z(\hat{H}))\right]^2 > 0 \tag{A10}$$

where $\hat{H}$ is the optimal number of hours spent on home production at wage rate W. Differentiating the left-hand side of (A6) with respect to H we get

$$\frac{\partial^2 Z}{\partial X_Z \partial H}(H, \psi_Z(H)) + \frac{\partial^2 Z}{(\partial X_Z)^2}(H, \psi_Z(H))\frac{\partial \psi_Z}{\partial H}(H) = 0 \quad \text{on } \mathcal{H}$$

Together with (A10) and (A8) this gives

$$\frac{\partial^2 Z}{(\partial H)^2}(\hat{H}, \psi_Z(\hat{H})) + g''(\hat{H})W + \frac{\partial^2 Z}{\partial X_Z \partial H}(\hat{H}, \psi_Z(\hat{H}))\frac{\partial \psi_Z}{\partial H}(\hat{H}) < 0 \tag{A11}$$

The partial maximization problem of an employed individual with production function F and no joint production is:

$$\max_{H\in\mathcal{H};X_Z\geq 0} Z(H,X_Z)+\int_{H_0}^{H}\frac{g'(\tilde{H})}{1-g'(\tilde{H})}\frac{\partial Z}{\partial H}(\tilde{H},\psi_Z(\tilde{H}))d\tilde{H}-WH-X_Z \tag{A12}$$

The conditions for local strict concavity of the maximand in $(\hat{H},\psi_Z(\hat{H}))$ are that

$$\frac{\partial^2 Z}{(\partial X_Z)^2}(\hat{H},\psi_Z(\hat{H})) \text{ and}$$

$$\frac{\partial^2 Z}{(\partial X_Z)^2}(\hat{H},\psi_Z(\hat{H}))\left\{\frac{g''(\hat{H})}{1-g'(\hat{H})}W+\frac{1}{1-g'(\hat{H})}\frac{\partial^2 Z}{(\partial H)^2}(\hat{H},\psi_Z(\hat{H}))\right.$$
$$\left.+\frac{g'(\hat{H})}{1-g'(\hat{H})}\frac{\partial^2 Z}{\partial H\partial X_Z}(\hat{H},\psi_Z(\hat{H}))\frac{\partial\psi_Z}{\partial H}(\hat{H})\right\}-\left[\frac{\partial^2 Z}{\partial H\partial X_Z}(\hat{H},\psi_Z(\hat{H}))\right]^2$$

should be negative and positive, respectively. The first requirement is met by (A8). The second expression can be rewritten as:

$$\frac{\partial^2 Z}{(\partial X_Z)^2}(\hat{H},\psi_Z(\hat{H}))\frac{g'(\hat{H})}{1-g'(\hat{H})}\left\{g''(\hat{H})W+\frac{\partial^2 Z}{(\partial H)^2}(\hat{H},\psi_Z(\hat{H}))\right.$$
$$\left.+\frac{\partial^2 Z}{\partial H\partial X_Z}(\hat{H},\psi_Z(\hat{H}))\frac{\partial\psi_Z}{\partial H}(\hat{H})\right\}$$
$$+\frac{\partial^2 Z}{(\partial X_Z)^2}(\hat{H},\psi_Z(\hat{H}))\left\{\frac{\partial^2 Z}{(\partial H)^2}(\hat{H},\psi_Z(\hat{H}))+g''(\hat{H})W\right\}-\left[\frac{\partial^2 Z}{\partial H\partial X_Z}(\hat{H},\psi_Z(\hat{H}))\right]^2$$

From (A8), (A11) and (A10) this expression is seen to be positive. We now only have to verify that $\hat{H}$ is also a global optimum for (A12).

Condition (A11) guarantees that the right-hand term in (A5) is strictly decreasing on $\mathcal{H}$. From this it is easy to see that the integrand in (27) is strictly decreasing on (H_0,H^*), i.e.

$$\frac{g'(H^*)}{1-g'(H^*)}\frac{\partial Z}{\partial H}(H^*,\psi_Z(H^*))\leq Wg'(\hat{H})\leq\frac{g'(H_0)}{1-g'(H_0)}\frac{\partial Z}{\partial H}(H_0,\psi_Z(H_0)) \tag{A13}$$

The objective function of (A12) differs from that in (A7) by a term $Q(H)-Wg(H)$, where $Q(H)\stackrel{\text{def}}{=}F(H,X_Z)-Z(H,X_Z)$. From (A13) it follows that this term is monotonically increasing on $[0,H_0]$ and monotonically decreasing on $[H^*,T]$. On (H_0,H^*):

$$Q'(H)-Wg'(H)=g'(H)\left\{\frac{1}{1-g'(H)}\frac{\partial Z}{\partial H}(H,\psi_Z(H))-W\right\}$$

Table AI. Sample means and standard deviations (two-adult households)

	$N_m > 0$ $N_f > 0$	$N_m > 0$ $N_f = 0$	$N_m = 0$ $N_f > 0$	$N_m = 0$ $N_f = 0$
Number of observations	517	139	75	88
Home production male (min./day)				
Householdwork	43.6 (47.9)	47.2 (58.5)	84.1 (75.0)	80.9 (79.3)
Childcare	15.1 (38.0)	15.4 (36.1)	15.9 (41.3)	10.3 (34.8)
Repair and maintenance	36.3 (65.7)	39.3 (78.1)	63.2 (100.0)	62.4 (89.5)
Total (H_m)	94.9 (80.1)	101.9 (97.8)	163.2 (116.1)	153.6 (117.4)
Home production female (min./day)				
Householdwork	142.9 (86.3)	221.7 (118.6)	144.4 (95.8)	251.0 (117.2)
Childcare	29.7 (67.4)	57.0 (80.9)	15.4 (36.5)	7.0 (24.8)
Repair and maintenance	8.9 (30.8)	18.7 (50.7)	11.2 (36.2)	24.2 (53.4)
Total (H_f)	181.5 (108.2)	297.4 (146.3)	171.1 (104.7)	282.1 (140.2)
Male characteristics				
Age	42.3 (10.3)	43.5 (12.5)	55.5 (13.3)	67.3 (9.2)
Education (years)	11.2 (3.6)	11.0 (4.1)	9.6 (3.4)	7.8 (3.0)
Gross wage (sek per hour)	52.5 (23.8)	52.2 (39.3)	50.2 (24.1)	50.0 (31.3)
Non-labour income (1000 sek/yr)	6.8 (14.4)	10.5 (14.2)	60.3 (54.7)	59.5 (37.5)
Average tax rate (%)	35.1 (19.0)	33.1 (10.5)	30.1 (13.8)	27.9 (11.9)
Marginal tax rate (%)	52.8 (11.4)	50.2 (12.9)	43.6 (20.3)	40.6 (18.3)
Labourmarket status				
Employed	452	119	—	—
Self-employed	65	20	—	—
Unemployed	—	—	18	2
Not in the labour force	—	—	57	86
Female characteristics				
Age	39.7 (10.1)	41.0 (12.7)	49.9 (12.1)	64.2 (8.7)
Education (years)	11.0 (3.3)	10.2 (3.0)	9.0 (3.0)	7.7 (2.4)
Gross wage (sek per hour)	43.4 (24.3)	40.1 (17.6)	40.4 (11.7)	39.5 (5.3)
Non-labour income (1000 sek/yr)	7.0 (12.3)	17.2 (20.3)	10.1 (15.3)	31.0 (22.0)
Average tax rate (%)	30.0 (16.4)	16.8 (13.3)	28.8 (7.8)	20.6 (10.6)
Marginal tax rate (%)	42.8 (19.9)	24.9 (18.6)	42.1 (10.8)	30.5 (14.2)
Labourmarket status				
Employed	492	—	73	—
Self-employed	25	—	2	—
Unemployed	—	27	—	4
Not in the labour force	—	112	—	84
Household characteristics				
Homeowner (%)	79.8 (40.1)	68.3 (46.7)	62.7 (48.7)	61.4 (49.0)
Number of household members	3.5 (1.1)	3.4 (1.2)	2.9 (1.1)	2.4 (1.2)
Number of adults	2.3 (0.6)	2.2 (0.4)	2.2 (0.6)	2.1 (0.4)
Number of children younger than 7	0.36 (0.6)	0.58 (0.8)	0.20 (0.5)	0.03 (0.2)
Technology intensive (%)	16.6 (37.3)	13.7 (34.5)	8.0 (27.3)	5.6 (23.3)
Owner of leisurehouse (%)	24.8 (43.2)	0.2 (0.4)	33.3 (47.5)	0.3 (0.4)
Number of cars	1.2 (0.7)	1.2 (0.8)	1.1 (0.7)	0.8 (0.6)
Owns a boat (%)	32.9 (47.0)	23.7 (42.7)	25.3 (43.8)	19.3 (39.7)

which is monotonically decreasing, with value zero in $\hat{H}$. Therefore, $Q(H)$ attains its maximum value in $\hat{H}$ and the maximization problem (A12) has its unique global maximum in $\hat{H}$. The household production model with production function F and no joint production is admissible.

$Q(H)$ is linear on $[0, H_0] \cup [H^*, T]$ and—as we just learned—concave on (H_0, H^*). As a consequence, F is strictly concave on $[0, T] \times \mathbb{R}_+$ if Z is QED.

Table AII. Sample means and standard deviations (single adult households)

	Males		Females	
	$N > 0$	$N = 0$	$N > 0$	$N = 0$
Number of observations	83	39	100	64
Home production (min./day)				
Householdwork	56.4 (63.3)	101.7 (67.5)	105.0 (90.1)	183.3 (107.0)
Childcare	2.8 (15.1)	——	8.4 (36.2)	12.8 (63.4)
Repair and maintenance	32.2 (60.8)	45.8 (72.9)	11.0 (41.0)	10.7 (30.1)
Total (H)	91.4 (83.2)	147.5 (117.2)	124.3 (102.7)	206.8 (122.2)
Individual characteristics				
Age	40.3 (12.1)	55.7 (19.5)	39.0 (13.2)	58.4 (16.6)
Education (years)	11.0 (3.3)	8.3 (2.8)	11.2 (3.2)	8.9 (3.5)
Gross wage (sek per hour)	45.1 (17.7)	39.2 (8.1)	40.5 (11.7)	39.7 (8.1)
Non-labour income (1000 sek/yr)	10.5 (16.8)	43.8 (33.3)	12.2 (16.2)	42.8 (28.5)
Average tax rate (%)	33.4 (9.7)	24.4 (11.1)	31.3 (6.0)	23.6 (11.5)
Marginal tax rate (%)	51.7 (10.8)	34.7 (15.1)	47.6 (8.4)	33.6 (16.3)
Labourmarket status				
Employed	72	—	97	—
Self-employed	11	—	3	—
Unemployed	—	8	—	8
Not in the labour force	—	31	—	56
Household characteristics				
Houseowner (%)	41.0 (49.5)	43.6 (50.2)	27.0 (44.6)	26.6 (44.5)
Number of household members	1.42 (1.0)	1.15 (0.5)	1.82 (1.0)	1.28 (0.7)
Number of adults	1.24 (0.6)	1.13 (0.5)	1.33 (0.7)	1.14 (0.4)
Number of children younger than 7	0.05 (0.3)	0.03 (0.2)	0.09 (0.3)	0.03 (0.2)
Technology intensive (%)	2.4 (15.4)	——	1.0 (10.0)	1.6 (12.5)
Owner of leisurehouse (%)	20.5 (40.6)	5.1 (22.3)	11.0 (31.4)	9.4 (29.4)
Number of cars	1.04 (1.1)	0.64 (0.7)	0.50 (0.6)	0.25 (0.47)
Owns a boat (%)	26.5 (44.4)	5.1 (22.3)	7.0 (25.6)	4.7 (21.3)

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